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Evolution of Inventory Management: From Classical to Fuzzy and Extended Fuzzy Models

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Abstract: Inventory management is a basic area of modern supply chain and operations research. The main aim is to optimize the total cost of inventory and maintain an optimal service level. There has been significant progress in inventory models over the years. Initial studies developed basic deterministic models, such as the Economic Order Quantity (EOQ) model, which considered fixed demand, cost, and lead time. Later, fuzzy inventory models were developed to handle uncertainties by using fuzzy numbers for demand, cost, and breakdown, and by using uncertainty and linguistic terms in decision-making. Recently, extended fuzzy models have been developed, which include concepts such as Pythagorean fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, and neutrosophic sets. This review aims to describe the evolution of inventory models from classical to fuzzy models and review recent trends in extended fuzzy models to provide a guideline for future research in this area.

Keywords: *Uncertainty, Inventory management, Supply chain Management, Holding cost.*

1. Introduction

Inventory control is a fundamental process of managing goods flow within a business. The primary objective of inventory control is to reduce the total cost of inventory and provide the right amount of product at the right time. Over the years, the process of managing goods flow has undergone significant changes. The process of managing goods flow began with a simple approach such as Economic Order Quantity (EOQ), which considers demand, cost, and lead time. However, the concept of supply chain management is a relatively new approach that considers the entire process of goods flow, money flow, and information flow between four key participants is suppliers, manufacturers, retailers, and customers. The flow of goods passes through various supply channels, which can be explained as follows Fig.1.

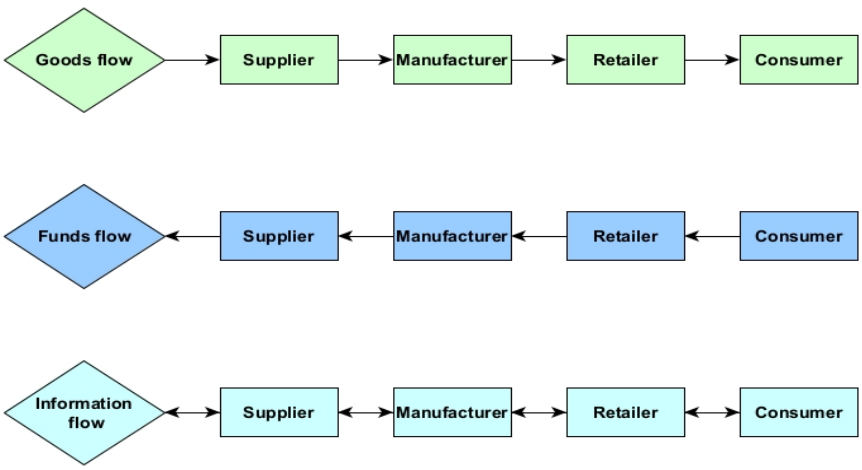

Fig.1: *Basic supply chain flow*

In Fig.1. above, the flow of products, financial assets, and information between the key parties in the supply chain, namely the supplier, the manufacturer, the retailer, and the consumer, is shown. First, the flow of products from the supplier to the manufacturer and finally to the consumer takes place. Second, the flow of financial assets from the consumer to the retailer, then from the retailer to the manufacturer, and finally from the manufacturer to the supplier takes place. Third, the flow of information such as orders, demand, and feedback from the consumer to the retailer, then to the manufacturer, and finally to the supplier takes place.

The inventory problem can be simplified to designing a policy that answers two key questions:

1. What quantity should be ordered?
2. At what time should the order be placed?

Deciding how much to order means determining the order quantity at each replenishment. However, deciding when to order is more complex. Inventory systems may use periodic reviews, where orders are placed at fixed intervals such as weekly or monthly, or continuous reviews, where a new order is placed whenever the inventory level falls to a predetermined reorder point. Retail stores provide a common example: replenishment is periodic when items are ordered every week or month, and continuous when new stock is ordered as soon as the inventory drops below a specific level. The basic idea of an inventory model is to combine all relevant costs into one total cost function. This total inventory cost is usually expressed as the sum of four main components.

i.e., Total inventory cost = Purchasing cost + Setup cost + Holding cost + Shortage cost

Building on this foundational principle, recent studies have explored several advanced aspects of inventory management in dynamic and uncertain environments. Chen and Chao (2019) [1] study dynamic inventory control with stockout substitution and demand learning, while Chen et al. (2019) [2] develop learning-based methods that jointly handle pricing and additive manufacturing for spare parts. Mecheter et al. (2022) [3] review inventory control under lost sales and censored demand, and Dubey and Kumar (2023) [4] summarise recent trends and technological advances in inventory management. Flores et al. (2024) [5] focus on agricultural supply chains, showing how improved inventory practices support waste reduction and sustainability goals.

1.1. Motivation and Objective

Technology is evolving rapidly, and this changes the nature of inventory management. The current scenario presents several applications that call for attention, including the evolution of complexity, customer behaviour, and increasing focus on sustainability.

This review proposes the following objective:

- To provide an in-depth analysis of technological developments in inventory management in an uncertain environment.
- To examine the evolution of multi-echelon inventory optimization approaches and their practical applications.
- To assess the integration of sustainability principles into inventory management and the development of green supply chain practices.
- To identify current research gaps, implementation challenges, and future opportunities in the field.
- To provide practical recommendations for organizations, researchers, and policymakers based on the synthesis of existing literature and emerging trends.

The manuscript is divided into three parts. Section 1 introduces the basics of inventory management and the importance and aims of the research. Section 2 tackles the imprecision in inventory management, explaining the fuzzy inventory model and the neutrosophic inventory model, as well as the recent research done by different authors. Finally, the conclusion summarizes the main findings of the study.

2. Uncertainty in Inventory Management

Uncertainty in inventory management appears in a variety of forms, each of which poses different challenges and requires different management approaches. It is important to have a clear understanding of the different types of uncertainties to develop effective methods for computation. This section describes the major types of uncertainties that organizations face in inventory management.

❖ Demand Uncertainty

Uncertainty in demand is a fundamental challenge in managing inventories. It is caused by seasonality and swings in demands and economic tides, as well as competitive action and the various stages of a product's life cycle. Mix in the effects of weather, competitors' actions, overall economic trends, and the process toward obsolescence, and demand planning becomes more complicated. Moreover, demand uncertainty instabilities in material availability, lead-time and quality, and costs is reignited in contemporary global value chains as a prominent feature in our highly interwoven and globally networked markets and suppliers.

❖ Supply Uncertainty

The uncertainty of supply is related to the variability of the timing, quality, cost, and availability of materials, even when material supplies are expected. This type of uncertainty is an increasingly difficult problem for supply chains around the world. The factors that have influenced this type of uncertainty include reaching the capacity limits of suppliers, quality disruptions, unstable financial situations, geopolitical events such as wars or sanctions, delays in shipping, as well as logistics, which can increase lead times and variability. Other factors include differences in product quality, price variability, currency variations, and increased costs of living.

❖ Environmental and External Uncertainties

External environmental factors bring in their own set of uncertainties in the management of inventory. Organizations need to adapt to these external factors. External factors could be government regulations, natural disasters, climate change, technology disruptions, and global health emergencies. Examples of external factors include natural disasters, technology disruptions, and global health emergencies such as the COVID-19 pandemic.

In this context, some of the important uncertainties that have a major effect on inventory management have been described. To deal with these uncertainties, Zadeh [6] proposed fuzzy theory in 1965. The introduction of this theory brought a breakthrough, as it provides an effective framework for dealing with the uncertainties discussed above. The table 1 given below summarizes and discusses the contributions of various authors in the field of fuzzy inventory models.

Table 1: Review of Literature on Fuzzy Inventory Management

Authors	Years	Contributions
Berman et al. [7]	2019	A back-ordered inventory model with inflation in a cloudy fuzzy environment
Nayak et al. [8]	2019	A fuzzy inventory model for deteriorating items following a Weibull distribution under fully backlogged shortages.
Sharma et al. [9]	2021	An inventory model for reverse logistics that includes multiple production and remanufacturing batches, developed under a fuzzy (uncertain) environment.

Kumar et al. [10]	2021	A fuzzy reverse logistics inventory model for smart items using two warehouses of a retailer while considering carbon emissions.
Kumar and Paikray [11]	2022	A cost optimization inventory model for deteriorating items with a trapezoidal demand rate and fully backlogged shortages in both crisp and fuzzy environments.
Priyan et al. [12]	2022	A sustainable dual-channel inventory model with trapezoidal fuzzy demand and energy consumption.
Kuppulakshmi et al. [13]	2023	A fuzzy inventory model for imperfect items with price discounts and penalty maintenance costs.
Chaudhary et al. [14]	2023	A sustainable inventory model for defective items in a fuzzy environment.
Pakhira et al. [15]	2024	Developing a fuzzy logic-based inventory model that includes carbon emission costs and memory effects.
Mittal et al. [16]	2024	Optimizing inventory management: analysis of models using different fuzzy demand functions.

In Table 1, only fuzzy inventory management has been discussed. Similarly, various authors have proposed different models to handle uncertainty in inventory management, such as intuitionistic fuzzy sets introduced by Atanassov [17] in 1986 and Pythagorean fuzzy sets proposed by Yager [18] in 2013. However, in this paper, the focus is limited to fuzzy inventory and neutrosophic inventory models. In the following paragraph, the contributions of different researchers in neutrosophic inventory management are discussed.

Research on neutrosophic and fuzzy inventory systems has progressed steadily over the years. Surya and Mullai (2020) [19] first introduced a neutrosophic inventory model for deficient items with immediate returns, marking an early effort to address uncertainty using neutrosophic principles. Maissam Jdid (2023) [20] later examined important neutrosophic economic indicators for static inventory models without deficit, further strengthening the theoretical foundation. In 2024, a significant rise in related contributions can be observed. Surya et al. (2024) [21] developed a neutrosophic inventory system for decaying items with price-dependent demand, while Supakar et al. (2024) [22] applied the artificial bee colony algorithm to a green production-inventory model for deteriorating items in a neutrosophic fuzzy environment. During the same year, Dubey and Kumar (2024) [23] [24] [25] published several works, including a cost-effective neutrosophic inventory model, an inventory model with sensitivity analysis under uncertainty, and an optimization approach based on fuzzy extended principles. Additionally, Dubey et al. (2024) [26] provided a comprehensive survey on neutrosophic principles applied to inventory management problems. Collectively, these year-wise developments highlight the growing maturity and relevance of neutrosophic and fuzzy methodologies in modern inventory management.

Conclusion

In conclusion, the development of inventory control models represents the growing need to incorporate the issues of uncertainty and complexity in real-world inventory systems. Although the traditional deterministic models of inventory have established a theoretical foundation for inventory management, their limitations make them less applicable in uncertain environments. The development of fuzzy inventory models represented a breakthrough in dealing with imprecision and linguistic information in decision-making. More recently, the application of extended fuzzy models, such as intuitionistic, Pythagorean, hesitant, and neutrosophic sets, has further enhanced the ability of inventory models to deal with uncertainty, hesitation, and inconsistent information.

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